Let us now consider wave Propagation, at arbitrary frequencies, Parallel to the equilibrium Magnetic field. When $\theta = 0$, The eigenmode equ^n simplifies to

$$\begin{pmatrix} S-n^2 & -iD & 0 \\ iD & S-n^2 & 0 \\ 0 & 0 & P \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0. \qquad \text{— (1)}$$

One obvious way of Solving This equ^n is to have

$$P \simeq 1 - \frac{\Pi_e^2}{\omega^2} = 0 \qquad \text{— (2)}$$

with the eigenvector $(0, 0, E_z)$. This is just The electrostatic Plasma wave which we found Previously in an unmagnetized Plasma. This mode is longitudinal in nature, and Therefore Causes particles to oscillate Parallel to B_0. It follows That The Particles experience Zero Lorentz force due to the Presence of the equilibrium magnetic field, with The result That This field has no effect on The Mode dynamics.

The Other two Solutions to equ^n (1) are obtained by Setting The 2×2 determinant involving The x^- and y^- components of The electric field to Zero. The first wave has the dispersion relation.

$$n^2 = R \simeq 1 - \frac{\Pi_e^2}{(\omega + \Omega_e)(\omega + \Omega_i)} \qquad \text{— (3)}$$

and The eigenvector $(E_x, iE_x, 0)$. This is evidently a right-handed Circularly Polarized Wave.

The second wave has the dispersion relation

$$n^2 = L \simeq 1 - \frac{\Pi_e^2}{(\omega - \Omega_e)(\omega - \Omega_i)}, \qquad (4)$$

and the eigenvector $(E_x, -i E_x, 0)$. This is evidently a left-handed circularly polarized wave. At low frequencies (i.e, $\omega \ll \Omega_i$), both wave tend to the Alfven wave found previously. Note that the fast and slow Alfven waves are indistinguishable for parallel propagation. Let us now examine the high-frequency behaviour of the right and left-handed waves.

For the right-handed wave, it is evident, since Ω_e is -ve, that $n^2 \to \infty$ as $\omega \to |\Omega_e|$. This resonance, which corresponds to $R \to \infty$, is termed the electron cyclotron resonance. At the electron cyclotron resonance the transverse electric field associated at the same velocity, and in the same direction, as electrons gyrating around the equilibrium magnetic field. Thus the electrons experience a continuous acceleration from the electric field, which tends to increase their perpendicular energy. It is therefore, not surprising that right-handed waves, propagating parallel to the equilibrium magnetic field, and oscillating at the frequency Ω_e, are absorbed by electrons.

When ω is just above $|\Omega_e|$, we find that Π^2 is -ve and so there is no wave propagation in this frequency range. However, for frequencies much greater than the electron cyclotron <u>or</u> plasma frequencies, the solution to eqn (3) is approximately $n^2 = 1$. In other words $\omega^2 = k^2 c^2$. The dispersion relation of a right-handed vacuum electromagnetic wave. Evidently, at some frequency above $|\Omega_e|$ the solution for Π^2 must pass through zero and become positive again. Putting $\Pi^2 = 0$ in equ^n (3) we find that the equ^n reduces to

$$\omega^2 + \Omega_e \omega - \Pi_e^2 \simeq 0. \quad \text{— (5)}$$

assuming that $V_A \ll c$. The above equ^n has only one +ve root, at $\omega = \omega_1$ where

$$\omega_1 \simeq |\Omega_e|/2 + \sqrt{\Omega_e^2/4 + \Pi_e^2} > |\Omega_e|. \quad \text{— (6)}$$

Above this frequency, the wave propagates once again.

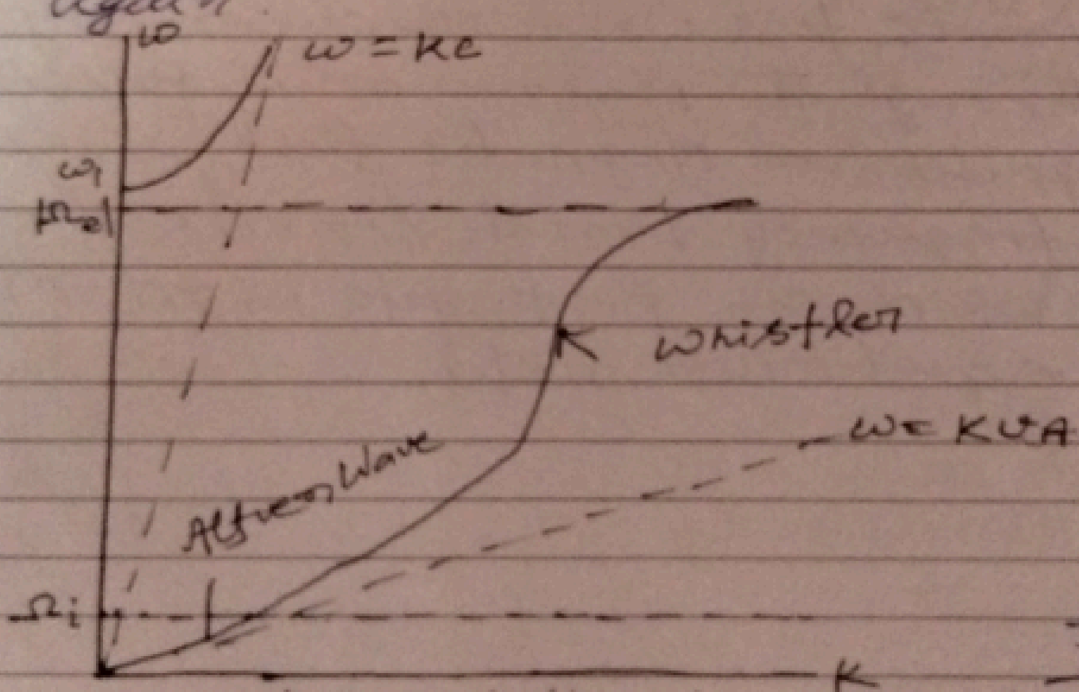

fig-(1)

Dispersion relation for a ~~...~~ right handed wave propagating parallel to the magnetic field in a magnetized plasma.

The dispersion curve for a right-handed wave propagating parallel to the equi-

librium magnetic field is sketched in figure.
The continuation of the Alfven wave above the ion cyclotron frequency is called the electron cyclotron wave or sometimes the whistler wave. The latter terminology is prevalent in ionospheric and space plasma physics contexts. The wave which propagates above the cutoff frequency, ω_1 is a standard right handed circularly polarized electromagnetic wave, somewhat modified by the presence of the plasma. Note that the low frequency branch of the dispersion curve differs fundamentally from the high-frequency branch, because the former branch corresponds to a wave which can only propagate through the plasma in the presence of an equilibrium magnetic field, whereas the high-frequency branch corresponds to a wave which can propagate in the absence of an equilibrium field.